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MA 495/595
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Homework 2

Due January 25th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. There are some hints on the second page.

1. Consider the traffic equation $u_t + (1-2u)u_x = 0$ with intial condition $u(0,x) = h(x)$, $h(x) = \max(0, 1-a|x|)$, where $a > 0$.
   (a) Find the characteristics $x(t)$ and sketch them. Use this to find T in terms of a, as large as possible, such that the solution $u(t,x)$ exists for all x and for $0 \leq t < T$.
   (b) Write a formula for the solution $u(t,x)$ for these values of t in the form

$$u(t,x) = \begin{cases} h(\cdots), & x \geq -t, \\ h(\cdots), & x \leq -t, \end{cases}$$

   where the $\cdots$ are filled in with an explicit function of a, x and t. Sketch the graph of $u(t,x)$ for a small positive value of t.
2. Borthwick Exercises 3.6 and 3.7.

Hints:

1(a). Follow Example 3.9. The sketch looks like Figure 3.7 but shifted.

1(b). The $\cdots$ are both fractions where the numerator is $x + t$ and the denominator is a simple function of a and t. The graph of $u(t, x)$ is obtained from $h(x)$ by stretching the graph horizontally on one side of $x = 0$, squeezing it horizontally on the other side, and then shifting.

3.6. This is also similar to Example 3.9.